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US NAVAL OBSERVATORY EPHEMERIDES OF THE LARGEST ASTEROIDS

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ABSTRACT

A new set of ephemerides for 15 of the largest asteroids has been produced for use in the Astronomical Almanac. The ephemerides cover the period from 1800 through 2100. The internal uncertainty in the mean longitude at epoch, 1997 December 18, ranges from 0″.05 for 7 Iris through 0″.22 for 65 Cybele, and the uncertainty in the mean motion varies from 0″.02 per century for 4 Vesta to 0″.14 per century for 511 Davida. This compares very favorably with the internal errors for the outer planets in recent Jet Propulsion Laboratory planetary ephemerides. However, because the asteroids have relatively little mass and are subject to perturbations by other asteroids, the actual uncertainties in their mean motions are likely to be a few tenths of an arcsecond per century. As part of the improvement to the ephemerides, new masses and densities were determined for 1 Ceres, 2 Pallas, and 4 Vesta, the three largest asteroids. These masses are as follows: Ceres = $(4.39 \pm 0.04) \times 10^{-10}\ M_{\odot}$, Pallas = $(1.59 \pm 0.05) \times 10^{-10}\ M_{\odot}$, and Vesta = $(1.69 \pm 0.11) \times 10^{-10}\ M_{\odot}$. The mass for Ceres is smaller than most previous determinations of its mass. This smaller mass is a direct consequence of the increase in the mass determined for Pallas. The densities found for these three asteroids are 2.00 ± 0.03 g cm^{-3} for Ceres, 4.2 ± 0.3 gm cm^{-3} for Pallas, and 4.3 ± 0.3 g cm^{-3} for Vesta. The density for Ceres is somewhat greater than that found for the taxonomically similar 253 Mathilde.

Key words: celestial mechanics, stellar dynamics — minor planets, asteroids

1. INTRODUCTION

The Astronomical Almanac has published ephemerides for 1 Ceres, 2 Pallas, 3 Juno, and 4 Vesta since its edition for 1953. Historically, these four asteroids have been observed more than any of the others. Even in modern times few asteroids have had more attention paid to them. Ceres, Pallas, and Vesta deserve such attention because they are the three most massive asteroids, the source of significant perturbations of the planets, and among the brightest main belt asteroids, which makes them prime targets for photometric and spectroscopic studies aimed at understanding their composition. They are also the largest asteroids in linear size, so they have been the subjects of diameter determinations by groups such as Millis et al. (1987), Lambert (1985), Magnuson (1986), Drummond & Cocke (1988), and Thomas et al. (1997).

The ephemerides currently published in the Astronomical Almanac are based on the dated work of Duncombe (1969). These ephemerides extend only until 2000 January 7. As a result, a new set of ephemerides is needed.

Interest in the asteroids has increased significantly over the past several years for many reasons, such as searching for clues to the origin and primordial composition of the solar system, the chaotic dynamics of small solar system bodies, and the potential of asteroid collisions with Earth. The asteroids are also a source of significant perturbations of the major planets. DE200, the JPL planetary ephemerides currently used in the Astronomical Almanac, was constructed using perturbations from five asteroids (Standish 1990), while more recent planetary ephemerides such as DE403 (Standish et al. 1995) and DE405 (E. M. Standish, Jr. 1998, private communication) include perturbations from 300 asteroids. Thus, it was decided to include more than the traditional four asteroids in the production of new asteroid ephemerides. The criteria used to select a small sample of main belt asteroids for producing ephemerides were the following:

Asteroids over 300 km in diameter, presumably the most massive asteroids. These were chosen for future studies of their perturbations of the planets.

Asteroids with excellent observing histories and discovered before 1850. These were chosen to explore the accuracy limits to which current asteroid ephemerides can be determined.

Asteroids that were the largest in their taxonomic class.

A total of 15 asteroids met these criteria and are given in Table 1. The new ephemerides of these 15 asteroids make up the USNO/AE98 (US Naval Observatory Asteroid Ephemerides of 1998). The USNO/AE98 covers the period 1799 November 16 (JD 2378450.5) through 2100 February 1 (JD 2488100.5).

The construction of the ephemerides is discussed in the following sections. Section 2 covers the data used to determine the ephemerides and how the data were handled, § 3, the physical model used to integrate the ephemerides, § 4, the masses and densities of the largest asteroids. Section 5 discusses the resulting ephemerides, and § 6 looks at the residuals and places limits on the accuracy of the ephemerides.

2. DATA

An ephemeris is only as good as the observations that are used and the physical model used to generate it. The ephemerides of the asteroids are based on optical positions, like the ephemerides of the outer solar system planets. There are also two radar-delay observations of main-belt asteroids. The optical observations used in creating these ephemerides came from two data types: wide-angle data (mainly from transit instruments) and relative data (positions measured relative to nearby background stars).

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TABLE 1

ASTEROIDS SELECTED FOR EPHEMERIDES COMPUTATION

Asteroid	Diameter (km)	Observed before 1850	Largest in Class
1 Ceres	933[a]	X	X
2 Pallas	524[b]	X	X
3 Juno		X	
4 Vesta	530[c]	X	X
6 Hebe		X	
7 Iris		X	
8 Flora		X	
9 Metis		X	
10 Hygiea	407[d]	X	
15 Eunomia			X
16 Psyche			X
52 Europa	302[d]		
65 Cybele	310[e]		
511 Davida	326[d]		
704 Interamnia	317[d]		

[a] Diameter from Millis et al. 1987.
[b] Diameter from Drummond & Cocke 1988.
[c] Diameter from Thomas et al. 1997.
[d] Diameter from Tedesco 1992.
[e] Diameter from Tedesco 1989.

Most asteroid observations from the nineteenth century are of the wide-angle, fundamental catalog variety. The main sources of these observations are the Royal Greenwich Observatory; l'Observatoire de Paris; the Royal Observatory, Edinburgh; the Cambridge Observatory; the Royal Observatory, Cape of Good Hope; and the US Naval Observatory. These data were gathered from the annual publications of these observatories and Schubart (1976). An additional 4299 observations from other observatories, about 7% of the total observations, were gathered from the Astronomische Nachrichten (1823–1900). Early observations of Ceres and Pallas made at Palermo, Milan, and Seeburg were taken from Schubart (1976). In all, nineteenth-century wide-angle observations were gathered from 39 observatories.

Aside from Ceres, Pallas, Juno, and Vesta, very few wide-angle observations were made of asteroids from 1901 until 1985. The wide-angle asteroid observations for the twentieth century used here were provided by the Royal Greenwich Observatory, the Cape Observatory, the US Naval Observatory, the Carlsberg meridian circle (Carlsberg Meridian Catalog 1984–1995), and the Université de Bordeaux transit circle (Minor Planet Center 1997).

Another potential source of wide-angle data was the *Hipparcos* astrometric satellite. *Hipparcos* asteroid observations were examined for inclusion in the ephemerides. However, the *Hipparcos* observations are one-dimensional observations made along a great circle at an arbitrary inclination with respect to the celestial equator. The span of the *Hipparcos* mission was only 3.3 yr, so the usefulness of the observations is limited despite the high one-dimensional accuracy of the observations. For this reason the *Hipparcos* observations were not used.

The source for most of the relative observations used was the Minor Planet Center (MPC, 1997). There are two advantages to using the MPC data rather than collecting them from their original sources. First, the data are collected in a single place, saving time. Second, unlike wide-angle data, which are reduced to apparent position of date, relative data are reduced to a standard epoch. The relative position observations were gathered using three different methods: micrometer measurements in the mid-nineteenth century, photographic plates from the late nineteenth century until about 10 years ago, and CCD observations during the last 15 years. These observations were originally reduced to the dynamical coordinate system at a variety of epochs using several different methods. They also use the positions of stars as published in a variety of catalogs with varying degrees of accuracy. As a result, only the most recent relative position observations are of an accuracy comparable to that of the wide-angle transit observations. The MPC has provided the transformation from the original epoch of publication to the J2000.0 epoch. A total of 35,575 relative observations from 131 observatories were included.

Herget (1947) required that all observations from 1940 onward submitted to the MPC be reduced to the B1950.0 coordinate system. Earlier observations have been collected by the MPC and converted to B1950.0 coordinates using the information available along with the published observations. In 1992 January, when the MPC switched over to the J2000.0 coordinate system (Marsden 1991b), the MPC converted the positions of all of the observations it had at that time from the B1950.0 to the FK5/J2000.0 coordinate system using the procedures given on pages B42 and B43 of the Astronomical Almanac, modified for use with solar system objects rather than stars (Marsden 1991a).

However, it has always been the responsibility of the observer to reduce the observations to the required coordinate system. Thus it was necessary to subject all observations, both relative and wide angle, to scrutiny to make sure that there were no significant errors in the data. First, observatories that did not contribute at least 0.5% of the observations of an asteroid were dropped unless the observations were from the first or last opposition observed, or from an opposition that had no other observations. The data were divided into groups based on the object being observed, the observatory making the observations, and the age of the observations. Root mean square (rms) errors for all of the data were determined by assigning a priori values for the rms error, computing preliminary ephemerides, and then adjusting the size of the rms error using the residuals. For large data sets, containing more than 500 observations for an observatory over a given time period, offsets in right ascension and declination were solved for. In all cases the time period chosen was short enough that time-varying terms were insignificant. Offsets in the right ascension and declination were applied to 70% of the relative observations and 88% of the wide-angle observations. Smaller data sets were examined for deviation from zero in the mean residuals. Overall, about 5% of all of the observations were rejected. About 90% of the rejected observations came from observatories that did not contribute enough observations. In all cases, the systematic errors were found to be less than $1''$ and less than one-fifth of the rms error for any group of observations. The number of observations used for each asteroid are given in Table 2.

One other source of relative astronomical data was R. C. Stone (1995–1997, FASTT observations, private communication) at the US Naval Observatory Flagstaff Station. The technique used for these high-precision observations is described in Stone (1997). These data were extremely valuable because they provided very high accu-

TABLE 2

DATA COVERAGE FOR THE ASTEROID EPHEMERIDES

Asteroid	First Opposition	Last Opposition	Total Oppositions	Obs. in R.A.	Obs. in Decl.	Total Obs.
1 Ceres	1801	1996	139	9229	9031	9354
2 Pallas	1802	1996	138	9068	8907	9205
3 Juno	1804	1996	124	7617	7481	7751
4 Vesta	1807	1996	131	10324	10087	10475
6 Hebe	1847	1997	93	4701	4521	4737
7 Iris	1847	1997	85	4478	4279	4547
8 Flora	1847	1995	82	2190	1898	2247
9 Metis	1848	1995	68	1982	1724	2033
10 Hygiea	1849	1996	87	2009	1949	2035
15 Eunomia	1851	1996	70	1583	1357	1610
16 Psyche	1852	1997	80	1590	1526	1620
52 Europa	1858	1996	72	1145	1123	1156
65 Cybele	1861	1996	78	729	731	736
511 Davida	1903	1996	64	671	673	677
704 Interamnia	1910	1996	53	1392	1396	1398

racy data at the most recent opposition of the asteroids, providing a solid anchor point for the modern end of the asteroid observations.

Radar data have the potential of being the most useful of all the data types because of their high precision. A good radar observation will give the distance of a body within a couple of kilometers. The largest unknown is the apparent position of the center of reflection with respect to the center of mass. Ostro (1993) shows that until the recent upgrade of the Arecibo radio telescope, radar observations of all but the largest main belt asteroids have been impossible. To date, the only time-delay–Doppler observations published for the asteroids whose ephemerides are determined here are two observations of Iris by Ostro et al. (1991). These data were included in the ephemeris of Iris. Because the amount and time span of other data on Iris are large and the uncertainty in the time delay was rather large (42 and 24 km), the contribution of the radar data to the ephemerides was small. The effect on the orbital parameters was about 0.01 σ in the final solution.

3. PHYSICAL MODEL

The Planetary Ephemeris Program, PEP, is the software used for generating the asteroid ephemerides (Ash 1965). PEP is a high-accuracy program capable of generating ephemerides using complicated physical models, comparing the results to many different observation types, adjusting designated parameters, and then producing a new set of ephemerides. PEP can iterate the ephemerides until a desired level of convergence in the model parameters is reached. Standish (1987) compared a set of PEP-generated ephemerides with similar JPL ephemerides and found the differences between those ephemerides were less than their uncertainties. In addition to adjusting physical model parameters, PEP can adjust such parameters as catalog corrections in right ascension and declination, electronic delay biases in delay-Doppler observations, and corrections in the location of observatories. The final ephemerides were integrated using PEP's Adams-Moulton integrator with a step size of 2 days. The epoch of integration was 1997 December 18 (JD 2450800.5).

The planetary positions and masses used for perturbation of the integrated asteroid positions and determination of the $O-C$ values were taken from the JPL ephemeris DE405.

Asteroid perturbations are the largest source of incompletely modeled perturbations of the planets, especially Mars and the Earth-Moon barycenter. Williams (1984) shows no less than seven asteroids capable of making periodic perturbations of more than a kilometer in Mars's position. The largest asteroid, Ceres, has only 0.13% the mass of Mars and is located within the asteroid belt itself. Hence, it and the other asteroids are much more sensitive to the perturbations of the asteroids than are the planets. Thus, to achieve high accuracy, the physical model for the asteroid ephemerides must include perturbations by other asteroids.

The perturbing asteroids included in the ephemeris of each asteroid are given in Table 3. The mass for Interamnia is the mean of two values determined by Landgraff (1992). The mass for Davida is the estimate used by Viateau & Rapaport (1997). The mass for Eunomia is taken from Hilton (1997). The masses for Juno and Psyche are estimated from their Tedesco (1989) diameters and an assumed density of 3 g cm^{-1}. The masses of Ceres, Pallas, and Vesta are determined contemporaneously with the ephemerides.

TABLE 3

PERTURBING ASTEROIDS USED FOR EACH ASTEROID EPHEMERIS

Asteroid	Perturbing Asteroids
1 Ceres	Pallas, Vesta
2 Pallas	Ceres, Vesta
3 Juno	Ceres, Pallas, Vesta, Psyche, Davida
4 Vesta	Ceres, Pallas
6 Hebe	Ceres, Pallas, Vesta
7 Iris	Ceres, Pallas, Vesta
8 Flora	Ceres, Pallas, Vesta
9 Metis	Ceres, Pallas, Vesta
10 Hygiea	Ceres, Pallas, Vesta
15 Eunomia	Ceres, Pallas, Vesta, Davida
16 Psyche	Ceres, Pallas, Juno, Vesta
52 Europa	Ceres, Pallas, Vesta
65 Cybele	Ceres, Pallas, Vesta
511 Davida	Ceres, Pallas, Vesta, Eunomia, Interamnia
704 Interamnia	Ceres, Pallas, Vesta, Davida

TABLE 4

Masses and Densities of the Largest Asteroids

Asteroid	Type	Mass (10^{-10} $M_\odot$)	Volume (10^7 km^3)	Density (g cm^{-3})	Previous Mass (10^{-10} $M_\odot$)
1 Ceres	G	4.39 ± 0.04	43.7 ± 0.5	2.00 ± 0.03	4.71 ± 0.05[a]
2 Pallas......	B	1.59 ± 0.05	7.6 ± 0.4	4.2 ± 0.3	1.4 ± 0.2[b]
4 Vesta	V	1.69 ± 0.11	7.8 ± 0.3	4.3 ± 0.3	1.5 ± 0.3[b]

[a] Mass from Viateau 1995.
[b] Mass from Standish & Hellings 1989.

The final masses determined in a simultaneous solution are shown in Table 4 and discussed in § 4.

4. ASTEROID MASSES

The masses for the three largest asteroids, Ceres, Pallas, and Vesta, were determined from mutual perturbations. Table 5 gives some of the preliminary masses determined using various asteroids as perturbed bodies. Masses were determined both individually and simultaneously. Aside from the effect of the mass of Pallas on the mass determined for Ceres, discussed in the next subsection, the masses determined did not change significantly. The final masses in Table 4 were determined in a simultaneous solution using the perturbed asteroids in the table. Using other perturbed asteroids did not change the masses significantly, nor did they reduce the uncertainty in the derived mass of the perturbing asteroid.

4.1. *Ceres*

Figure 1 shows the record of recent determinations of the mass of Ceres as open circles. All of these determinations include Pallas in the gravitational model. Triangles are preliminary mass determinations for the ephemerides calculated here using Vesta and Juno as the perturbed asteroid including the historic mass for Pallas, approximately 1.2×10^{-10} $M_\odot$. The squares represent preliminary masses for Ceres using Pallas, Juno, and Vesta as the perturbed asteroid, with a simultaneous determination for the mass of Pallas, approximately 1.6×10^{-10} $M_\odot$. The final mass for Ceres from this study is a filled circle. The symbols for masses determined by other authors are open, and the symbols for the masses determined here are filled. The final mass of Ceres is significantly smaller than most modern estimates of its mass. The one similar previous Ceres mass determination is that of Kuzmanoski (1995) where the author treated the encounter of 203 Pompeja with Ceres using an impulse approximation.

TABLE 5

Preliminary Masses of the Largest Asteroids

Perturbing Asteroid	Perturbed Asteroid(s)	Mass (10^{-10} $M_\odot$)
Ceres	Juno[a,b]	4.69 ± 0.27
	Vesta[a,b]	4.82 ± 0.18
	Pallas[a,b]	4.37 ± 0.07
	Juno[a,c]	4.15 ± 0.27
	Vesta[a,c]	4.50 ± 0.18
	Pallas & Vesta[d]	4.35 ± 0.05
	Pallas & Vesta[e]	4.39 ± 0.04
Pallas	Ceres[b]	1.57 ± 0.06
	Ceres[d]	1.60 ± 0.04
	Ceres[e]	1.59 ± 0.05
Vesta	Ceres[b]	1.52 ± 0.15
	197 Arete	1.58 ± 0.11
	Ceres[d]	1.52 ± 0.09
	Ceres[e]	1.69 ± 0.11

[a] Mass of Pallas was 1.08×10^{-10} $M_\odot$.
[b] Nineteenth-century data from Royal Greenwich Observatory and Schubart 1976 only. Twentieth-century data from Royal Greenwich Observatory and US Naval Observatory only.
[c] Mass of Pallas was 1.57×10^{-10} $M_\odot$.
[d] Source of planetary ephemerides was DE200.
[e] Source of planetary ephemerides was DE405.

Why are these masses for Ceres so dependent on the mass of Pallas? The answer is that a degeneracy exists between the masses of Pallas and Ceres when fitting observations to an orbit over a limited period of time. This degeneracy results from semimajor axes of Ceres and Pallas that are very similar (the synodic period of Ceres and Pallas, based on the Williams 1989 proper semimajor axes, is 2000 yr) and a separation in mean longitude that has increased from approximately 1° at the time of discovery of Pallas to only about 43° in 1997 December. As a result, if the mass of Pallas is fixed at a wrong value, misattributed perturbations are propagated into the mass of Ceres. Figure 2 shows the history of the determinations of the mass of Pallas. The two most recent open circles are preliminary masses shown in Table 5. The first preliminary mass determination was made using approximately half the data in the final solution. The second preliminary mass determination was made using DE200, a different ephemeris for the perturbing planets. All three mass determinations agree with a mass for Pallas that is about 1.5 σ greater than the two most recent values using Ceres as the perturbed body; however, it is in agreement with the mass determined using the *Viking* lander data (Standish & Hellings 1989), the triangle. As Figure 1 shows, using the historic value for the mass of Pallas brings the

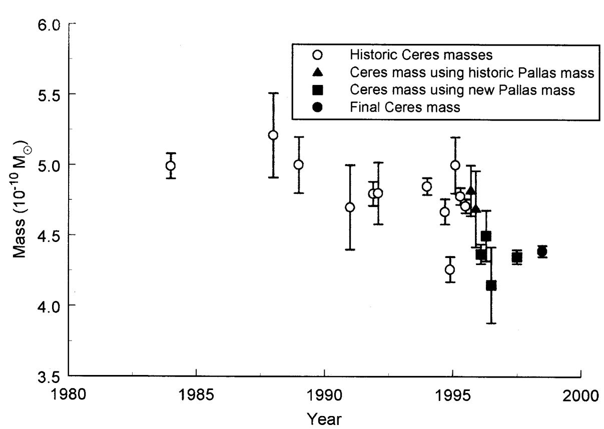

Fig. 1.—Recent mass determinations of 1 Ceres

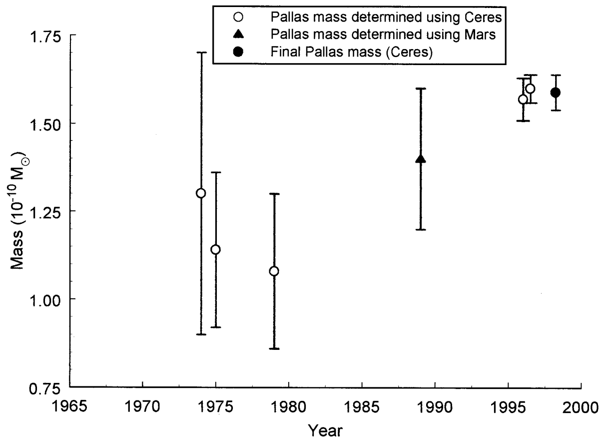

FIG. 2.—History of mass determinations of 2 Pallas

mass of Ceres into perfect agreement with most of the recent determinations of the mass of Ceres. Thus it is the difference in the mass determined for Pallas that changes the mass determined for Ceres. In the case of Kuzmanoski (1995), the act of treating the encounter as an impulse allowed the author to look only at the immediate effect of Ceres on Pompeja, thus ignoring the long-term effect of Pallas.

Ceres's orbit is mildly eccentric (0.097) and inclined 9°.7 to the ecliptic, while Pallas's orbit is more eccentric (0.180) and inclined 35°.7 to the ecliptic. Hence Ceres and Pallas are physically close only at the nodes of their orbits, even though they have similar mean distances and mean longitudes. This does not affect the degeneracy in determining the masses of Ceres and Pallas, however, because the observed effect is the average perturbation of Pallas over several orbits.

4.2. *Pallas*

Since an accurate determination of the mass of Ceres depends upon determining the mass of Pallas accurately, the accuracy of the mass of Pallas needs to be addressed. Table 5 shows that the mass determined here is robust. As long as the data set covered the full time span of the observations of Ceres and Pallas, the mass did not change significantly. Nor did it change significantly when the mass of Pallas was solved for alone or simultaneously with the masses of Ceres and Vesta. Finally, the 1 σ error is only 20% that of previous mass estimates. The mass of Pallas is not based on a single encounter with Ceres but on a series of close encounters that occurred in the years shortly after the discovery of Pallas early in the nineteenth century. Hence, the mass determined is most sensitive to the oldest, least accurate data. These data also have the greatest chance of containing unmodeled systematic errors. The possibility of systematic errors is reduced by using as many sources as possible, but this does not guarantee their elimination. Thus, confirmation of the mass of Pallas using another technique or a different perturbed asteroid is desirable.

The only existing alternative technique is that of Standish & Hellings (1989). Using this technique the masses for Ceres, Pallas, and Vesta are determined using the *Viking* lander ranging data. Just as the mass of Pallas found here is about 1 σ above the Standish & Hellings mass, the mass for Ceres is about 1 σ smaller than the Standish & Hellings mass, which reflects the degeneracy in determining their masses. However, Standish et al. (1995) reverted to a lower mass for Pallas for DE403 without explanation.

Finding other perturbed asteroids is difficult because Pallas is in a highly inclined, eccentric orbit, which reduces the number of chance close encounters. Those close encounters with Pallas that do occur are usually at high velocity, which reduces the size of the perturbation. The best candidate found so far is 2495 Noviomagnum, which encountered Pallas on 1991 January 1 at a minimum distance of 0.036 AU (Hilton, Seidelmann, & Middour 1996). Unfortunately, fitting to existing observations with Pallas as a perturbing body changes its right ascension by only 0".12 20 yr after the encounter when compared to an ephemeris generated without Pallas as a perturbing body.

4.3. *Vesta*

Figure 3 shows the history of the determination of masses for Vesta. Along with the final mass for this study (*filled circle*), there are two preliminary masses using Ceres and 197 Arete as the perturbed asteroid. The four masses on the left-hand side of the figure were determined by other authors, and the four masses determined here are on the right-hand side of the figure. All of them are in good agreement with other recent previous determinations. As with Ceres and Pallas, the mass of Vesta determined here has a significantly smaller uncertainty than the mass determination from the *Viking* lander data, the most recent determination prior to this one. The smaller uncertainty is a reflection of the fact that although the *Viking* lander observations are orders of magnitude more accurate than optical observations, the size of the perturbations of the asteroids on Mars is much smaller, and the *Viking* lander data span only 5% of the period covered by the optical data.

The final masses here were determined using a simultaneous solution for the masses of all three asteroids. Masses for all three asteroids were determined using a variety of observation data sets and initial conditions. The masses of Ceres and Vesta were also determined using other perturbed asteroids. Aside from the dependence of the mass determined for Ceres on the mass used for Pallas, all of the masses are robust. The masses determined in all the preliminary determinations were within 1.5 σ of the final masses. Thus, the mass uncertainties are quite realistic.

4.4. *Asteroid Densities*

New masses for Ceres, Pallas, and Vesta also provide new estimates for their densities.

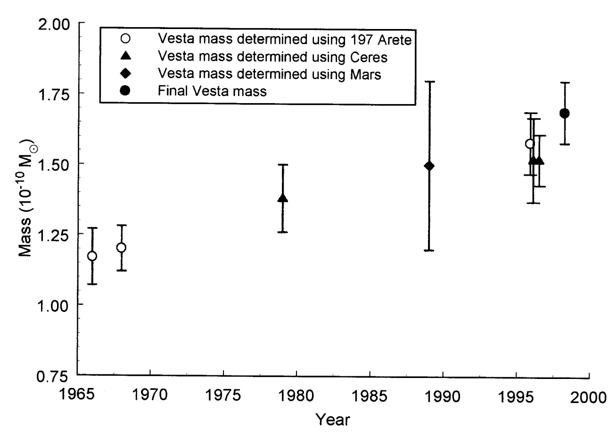

FIG. 3.—History of mass determinations of 4 Vesta

TABLE 6

UNCERTAINTY IN THE MEAN LONGITUDE AT EPOCH AND THE MEAN MOTION OF THE ASTEROIDS

Asteroid	Uncertainty in Mean Longitude (arcsec)	Uncertainty in Mean Motion (arcsec per century)
1 Ceres	0.05	0.023
2 Pallas	0.05	0.024
3 Juno	0.05	0.022
4 Vesta	0.05	0.017
6 Hebe	0.05	0.028
7 Iris	0.04	0.024
8 Flora	0.06	0.031
9 Metis	0.11	0.040
10 Hygiea	0.11	0.054
15 Eunomia	0.06	0.034
16 Psyche	0.11	0.076
52 Europa	0.16	0.109
65 Cybele	0.22	0.088
511 Davida	0.09	0.137
704 Interamnia	0.08	0.096

The volume of Ceres is based on the observation of a stellar occultation of BD $+8°471$ by Ceres (Millis et al. 1987) and *HST* images of Ceres by Merline et al. (1996). The mean radius is 470 ± 6 km, which gives a derived volume for Ceres of $(43.7 \pm 0.5) \times 10^7$ km^3. Combined with the mass of Ceres determined here gives a mean density of 2.00 ± 0.03 g cm^{-3}. This density is significantly greater than the density of the taxonomically similar 253 Mathilde (volume $80{,}000 \pm 12{,}000$ km^3 and density 1.3 ± 0.2 g cm^{-3}) (Veverka et al. 1997). Since Ceres is nearly 8500 times more massive than Mathilde, the difference in the density could be caused by greater compaction or a minimal amount of differentiation rather than a major difference in composition. Or, since Ceres is a G-type asteroid (considered a subtype of the C-type asteroids) while Mathilde is a C-type asteroid (Tholen 1989), compositional differences may account for the difference in density.

There are several determinations of Pallas's shape based on stellar occultations and speckle interferometry (Lambert 1985; Magnuson 1986; Drummond & Cocke 1988). All of these papers give similar results for the mean radius of Pallas. Using the Dummond & Cocke mean radius of 262 ± 13 km, the derived volume of Pallas is $(7.6 \pm 0.4) \times 10^7$ km^3. The density is 4.2 ± 0.3 gm cm^{-3}.

Thomas et al. (1997) have determined a mean radius for Vesta of 265 ± 5 km, which gives a volume of $(7.8 \pm 0.3) \times 10^7$ km^3. The derived density of Vesta is 4.3 ± 0.3 g cm^{-3}.

5. EPHEMERIDES

As described in § 3, the integration of the asteroid orbits, computation of the $O-C$ values, and adjustment of parameters to produce the ephemerides were carried out using PEP. The adjusted parameters in the solution were the osculating elements of the asteroids, the masses of Ceres, Pallas, and Vesta, and the catalog corrections for the 49 catalogs of observations that contributed 500 or more observations. The total number of adjusted parameters was 191. The epoch of integration was 1997 December 18 (JD 2450800.5).

The determination of catalog corrections had little effect upon the ephemerides generated. In all cases the adjustment was less than 1″, which was a factor of one-fifth or less of the rms uncertainty in the observations. Not adjusting the catalogs changed the initial conditions by only 10^{-6} AU in semimajor axis, 10^{-7} in eccentricity, and 0″.004 in the angular elements for Juno, the most extreme case. The change in apparent position on 1800 June 21.5 between the ephemeris with catalog corrections and that without catalog corrections is 0″.5 or about 17% of the rms uncertainty of observations from that time period.

The final ephemerides for all 15 asteroids covered the period 1799 November 16 (JD 2378450.5) through 2100 February 1 (JD 2488100.5) with a tabular interval of 2 days. The ephemerides give the position and velocity of each asteroid in equatorial rectangular coordinates on the mean equator and equinox of J2000.0. The positions are given in astronomical units (AU), and the velocities are in AU day^{-1}.

The osculating equatorial elements for the asteroids and their formal uncertainties at the epoch of integration for the final ephemerides listed are in the Appendix. From the uncertainty in the osculating elements, the uncertainty in mean longitude at the epoch of integration and the uncertainty in the mean motion of the asteroids is determined and is given in Table 6. Comparing the uncertainty in mean longitude and mean motion with Table 2 of Standish (1986) shows that the uncertainties in these ephemerides compare very favorably with the uncertainties of the outer planets in DE200. Thus, the formal errors give ephemerides that compare favorably with DE200.

What then are the realistic uncertainties in the ephemerides? The least-squares adjustment of parameters assumes that the physical model has no perturbations that are unaccounted for. All of these asteroids are in the main belt and have much smaller masses than the planets, however. Hence, unmodeled perturbations by other asteroids may cause departures from these ephemerides that are potentially significantly worse than the uncertainties given in Table 6. Juno, as discussed below, shows evidence of unaccounted for perturbations. Examination of Juno's ephemeris shows that the realistic uncertainties are, at most, a factor of 5 greater than the formal uncertainties.

6. RESIDUALS

Figure 4 shows the residuals in right ascension for Pallas and 65 Cybele. These residuals are representative of the residuals for all of the asteroids considered here. The bar shows the 3 σ rms of the residuals in each 20 yr era of observations. The number above the bar is the number of observations included in that era.

As expected, there is a reduction in the rms value of the residuals over time. Those residuals from the early nineteenth century have a 3 σ rms value of approximately 9″, while the 3 σ rms value for the last 20 yr of the twentieth century is approximately 2″. Between 1900 and 1960, the rms value in the residuals varies widely rather than decreasing with time. There are two reasons for the change in the rms value of the residuals for the early twentieth century. First, the early twentieth century is a period during which few astrometric observations were made of most asteroids. As a result, the statistics during this time period are poor. Second, during the early part of the twentieth century, several new observatories, such as Bucharest, Athens, Purple Mountain, Santiago–San Bernardo, and Madrid, began contributing observations that were significantly less

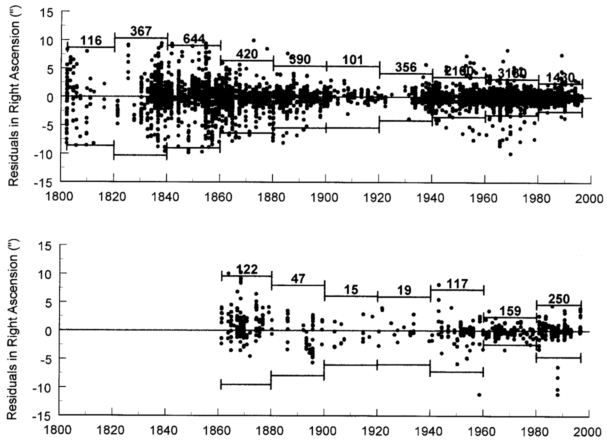

FIG. 4.—Residuals in right ascension for 2 Pallas (*top*) and 65 Cybele (*bottom*). The bar shows the 3 σ scatter in each 20 yr era of observations. The number above the bar is the number of observations included in that era.

accurate, at least initially, than those of older, established observatories. As described in § 2, the observations from these observatories were examined to make sure that they did not contain systematic errors.

6.1. *Juno*

Figure 5 shows the residuals in right ascension for Juno over its entire observed history. There is an obvious systematic departure of the residuals in right ascension before 1900. Table 7 shows that the mean residual in right ascension is large throughout the nineteenth century and is particularly large before 1840. The observations prior to 1830 are from two different observatories. Observations from both observatories show the same systematic drift in the residuals of Juno but do not show systematic drifts in the residuals of observations for Ceres, Pallas, or Vesta. Hence, this deviation is almost certainly caused by an encounter with an unmodeled asteroid during the mid-to-late nineteenth century.

A search for perturbing asteroids showed close approaches by Psyche and Davida and an approach with a minimum distance of several tenths of an AU by Interamnia. Other large asteroids known to come within a few tenths of an AU of Juno during the nineteenth century are 24 Themis, 87 Sylvia, and 216 Kleopatra. However, inclu-

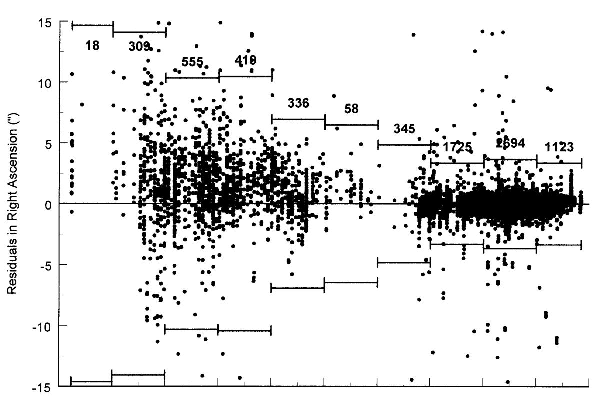

FIG. 5.—Residuals in right ascension for Juno from 1804 through 1996.

TABLE 7

MEAN RESIDUALS IN RIGHT ASCENSION FOR 3 JUNO

Time Period	Mean Uncertainty Right Ascension (arcsec)
1804–1820......	3.83
1821–1840......	1.75
1841–1860......	1.42
1861–1880......	1.81
1881–1900......	0.97
1901–1920......	1.15
1921–1940......	−0.18
1941–1960......	−0.08
1961–1980......	−0.15
1981–1996......	0.02

sion of these bodies as perturbers was able to account for only 20% of the runoff in the ephemeris of Juno. Thus, to remove the effect of the unmodeled perturbation, the observations prior to 1839, the point at which the deviation of the mean residuals becomes greater than $\frac{1}{3}$ σ, were removed from the final ephemeris. Removal of the observations prior to 1839 does not necessarily remove the entire effect of the unmodeled encounter; that can only be guaranteed by identifying the encounter and either including it or removing those observations prior to it. Removing the observations prior to 1839, however, should significantly lessen its effect.

The change in the initial conditions for Juno resulting from removing the data prior to 1839 was approximately 4 times the formal uncertainty in the initial conditions. This allows an estimate of the upper limits of the realistic uncertainties with respect to the formal uncertainties in the ephemerides. None of the other ephemerides show any obvious departures in their residuals like those of Juno. Thus it is unlikely that the effects of any unmodeled encounters for the other asteroids affect the ephemerides by more than a few tenths of an arcsecond.

As a further check on the effects of unmodeled encounters, C. Y. Hohenkerk (1997, private communication) compared the apparent positions produced by a preliminary ephemeris for Juno with those of the Duncombe (1969) ephemeris for Juno over the period from 1989 September 20 through 2000 January 20. There are deviations from the Duncombe ephemerides that are the result of perturbations not included in the Duncombe model. However, the deviations do not show any secular drift in the apparent positions. The largest deviation from the Duncombe ephemerides is the result of an encounter with 511 Davida. The maximum change in apparent position for this encounter was about 0″.75 in both right ascension and declination. Any secular change in mean position, however, was less than 0″.005 over this period. Thus, the maximum uncertainty in the mean motion is 0″.05 per century, twice the formal uncertainty. This analysis does not hold for the period prior to the unmodeled encounter, where the residuals indicate a change in the mean motion closer to 2″ per century.

The symmetric nature of the perturbation of Juno by Davida is unfortunate because there is no reliable determination of the mass of Davida, and the 2 yr span of the deviation in Juno's orbit is too short to provide a good determination from existing data. An attempt to determine the mass of Davida from its perturbation of Juno resulted in a mass with a formal uncertainty of about 200%.

7. CONCLUSIONS

A new set of ephemerides for 15 of the largest asteroids has been produced for use in the Astronomical Almanac. The ephemerides cover the period from 1800 through 2050.

A total of 59,258 optical and two radar observations were used to fit the ephemerides. Except for Juno, the observations cover the period from the discovery of the asteroid to the most recent opposition for which observations are available. Observations for Juno prior to 1839 were not included because the residuals in preliminary ephemerides indicated that Juno was affected by an unmodeled encounter with another large asteroid that resulted in a systematic drift between the ephemeris and the observations.

In improving the ephemerides, new masses were determined for Ceres, Pallas, and Vesta, the three largest asteroids. These masses are as follows: Ceres $= (4.39 \pm 0.04) \times 10^{-10}\ M_{\odot}$, Pallas $= (1.59 \pm 0.05) \times 10^{-10}\ M_{\odot}$, and Vesta $= (1.69 \pm 0.11) \times 10^{-10}\ M_{\odot}$. The mass for Ceres is smaller than most previous determinations of its mass. This smaller mass is a direct consequence of the increase in the mass determined for Pallas over previous determinations. The determination of the mass of Pallas from its effect on Ceres depends critically on the oldest, least accurate data. Hence it is desirable to make an independent determination of Pallas's mass. The mass determined by Standish & Hellings (1989) from Mars *Viking* observations is in accord with the mass determined here. There are no other known good asteroid candidates that are significantly perturbed by Pallas.

The densities for these three asteroids are 2.00 ± 0.03 g cm^{-3} for Ceres, 4.2 ± 0.3 g cm^{-3} for Pallas, and 4.3 ± 0.3 g cm^{-3} for Vesta. The density for Ceres is significantly greater than that of the taxonomically similar asteroid 253 Mathilde. This greater density may represent a greater compaction of the far larger Ceres, or it may represent a difference between the C-type asteroids (Mathilde) and the G-type asteroids (Ceres).

The internal accuracy of the ephemerides at epoch (1997 December 17; JD 2450800.5) ranges from 0.″05 for Iris through 0.″22 for Cybele, and the uncertainty in the mean motion varies from 0.″02 per century for Vesta to 0.″14 per century for Davida. This compares favorably with the internal errors for the outer planets in DE200. However, because the asteroids have relatively small mass and are subject to unmodeled perturbations by other asteroids, the actual uncertainties in their mean motions could be several times the internal error. Aside from Juno, there is no evidence of any large unmodeled perturbations. The ephemerides are estimated to be good to a few tenths of an arcsecond over the period covered by the observations.

The author would like to acknowledge the contribution of Rahim Taghizadegan for collecting the nineteenth-century observations of the asteroids from the Astronomische Nachrichten.

APPENDIX

OSCULATING ELEMENTS OF THE ASTEROIDS

Table 8 gives the osculating elements of the mean equator and equinox of J2000.0 for the asteroids at the epoch of integration, 1997 December 18 (JD 2450800.5).

TABLE 8

EQUATORIAL OSCULATING ELEMENTS FOR THE ASTEROIDS ON THE MEAN EQUATOR AND EQUINOX OF J2000.0

Asteroid	Element	Units	Value	Uncertainty
1 Ceres	Semimajor axis	AU	2.767837933	2×10^{-9}
	Eccentricity		0.07741186	2×10^{-8}
	Inclination	Degrees	27.143874	2×10^{-6}
	Ascending node	Degrees	23.390576	4×10^{-6}
	Argument of perihelion	Degrees	132.77714	1×10^{-5}
	Mean anomaly	Degrees	207.08207	1×10^{-5}
2 Pallas......	Semimajor axis	AU	2.773856966	2×10^{-9}
	Eccentricity		0.23233958	2×10^{-8}
	Inclination	Degrees	11.833838	2×10^{-6}
	Ascending node	Degrees	160.858815	9×10^{-6}
	Argument of perihelion	Degrees	323.02675	1×10^{-5}
	Mean anomaly	Degrees	194.831730	6×10^{-6}
3 Juno	Semimajor axis	AU	2.669481231	1×10^{-9}
	Eccentricity		0.25773186	2×10^{-8}
	Inclination	Degrees	10.876713	2×10^{-6}
	Ascending node	Degrees	11.70013	1×10^{-5}
	Argument of perihelion	Degrees	46.91848	1×10^{-5}
	Mean anomaly	Degrees	72.053979	2×10^{-6}
4 Vesta	Semimajor axis	AU	2.3607012365	7×10^{-10}
	Eccentricity		0.09035411	1×10^{-8}
	Inclination	Degrees	22.735663	2×10^{-6}
	Ascending node	Degrees	18.172807	4×10^{-6}
	Argument of perihelion	Degrees	237.07613	1×10^{-5}
	Mean anomaly	Degrees	138.499763	9×10^{-6}

TABLE 8—*Continued*

Asteroid	Element	Units	Value	Uncertainty
6 Hebe	Semimajor axis	AU	2.424774096	1×10^{-9}
	Eccentricity		0.20184319	1×10^{-8}
	Inclination	Degrees	15.512280	2×10^{-6}
	Ascending node	Degrees	38.815450	9×10^{-6}
	Argument of perihelion	Degrees	341.13194	1×10^{-5}
	Mean anomaly	Degrees	211.606826	7×10^{-6}
7 Iris	Semimajor axis	AU	2.384906313	1×10^{-9}
	Eccentricity		0.23063285	2×10^{-8}
	Inclination	Degrees	23.084891	2×10^{-6}
	Ascending node	Degrees	346.012381	6×10^{-6}
	Argument of perihelion	Degrees	57.998014	8×10^{-6}
	Mean anomaly	Degrees	214.061635	6×10^{-6}
8 Flora	Semimajor axis	AU	2.201299304	1×10^{-9}
	Eccentricity		0.15606734	3×10^{-8}
	Inclination	Degrees	21.982003	3×10^{-6}
	Ascending node	Degrees	14.813341	8×10^{-6}
	Argument of perihelion	Degrees	22.35365	1×10^{-5}
	Mean anomaly	Degrees	1.47253	1×10^{-5}
9 Metis	Semimajor axis	AU	2.386443361	2×10^{-9}
	Eccentricity		0.12115009	3×10^{-8}
	Inclination	Degrees	25.935090	3×10^{-6}
	Ascending node	Degrees	11.976524	9×10^{-6}
	Argument of perihelion	Degrees	63.84889	2×10^{-5}
	Mean anomaly	Degrees	352.35224	2×10^{-5}
10 Hygiea	Semimajor axis	AU	3.136204913	5×10^{-9}
	Eccentricity		0.11985212	4×10^{-8}
	Inclination	Degrees	24.617826	3×10^{-6}
	Ascending node	Degrees	351.002735	8×10^{-6}
	Argument of perihelion	Degrees	246.59314	2×10^{-5}
	Mean anomaly	Degrees	206.86683	2×10^{-5}
15 Eunomia	Semimajor axis	AU	2.644308703	2×10^{-9}
	Eccentricity		0.18701476	4×10^{-8}
	Inclination	Degrees	30.019997	4×10^{-6}
	Ascending node	Degrees	338.083505	7×10^{-6}
	Argument of perihelion	Degrees	50.17445	1×10^{-5}
	Mean anomaly	Degrees	296.45252	1×10^{-5}
16 Psyche	Semimajor axis	AU	2.921527397	6×10^{-9}
	Eccentricity		0.13756275	3×10^{-8}
	Inclination	Degrees	20.800688	3×10^{-6}
	Ascending node	Degrees	4.29608	1×10^{-5}
	Argument of perihelion	Degrees	15.45439	2×10^{-5}
	Mean anomaly	Degrees	188.69446	2×10^{-5}
52 Europa	Semimajor axis	AU	3.099221614	9×10^{-9}
	Eccentricity		0.10051921	5×10^{-8}
	Inclination	Degrees	19.562461	4×10^{-6}
	Ascending node	Degrees	17.54586	1×10^{-5}
	Argument of perihelion	Degrees	94.54815	3×10^{-5}
	Mean anomaly	Degrees	268.51717	3×10^{-5}
65 Cybele	Semimajor axis	AU	3.43189701	1×10^{-8}
	Eccentricity		0.10414563	7×10^{-8}
	Inclination	Degrees	20.251438	7×10^{-6}
	Ascending node	Degrees	4.19667	2×10^{-5}
	Argument of perihelion	Degrees	258.73968	4×10^{-5}
	Mean anomaly	Degrees	127.70237	4×10^{-5}
511 Davida	Semimajor axis	AU	3.17167262	1×10^{-8}
	Eccentricity		0.18235222	6×10^{-8}
	Inclination	Degrees	23.710076	5×10^{-6}
	Ascending node	Degrees	40.56063	1×10^{-5}
	Argument of perihelion	Degrees	49.29936	2×10^{-5}
	Mean anomaly	Degrees	48.08832	1×10^{-5}
704 Interamnia	Semimajor axis	AU	3.064446939	8×10^{-9}
	Eccentricity		0.14595014	4×10^{-8}
	Inclination	Degrees	31.363464	3×10^{-6}
	Ascending node	Degrees	325.795506	8×10^{-6}
	Argument of perihelion	Degrees	45.58638	2×10^{-5}
	Mean anomaly	Degrees	106.07912	2×10^{-5}

REFERENCES

Ash, M. E. 1965, Generation of Planetary Ephemerides on an Electronic Computer, Lincoln Laboratory, Tech. Report 391
Astronomische Nachrichten. 1823–1900, 1–150
Carlsberg Meridian Catalog. 1984–1995, Numbers 1–9 (Copenhagen Univ. Obs.; Royal Greenwich Obs.; Real Instituto y Observatorio de la Armada en San Fernando)
Drummond, J. D., & Cocke, W. J. 1988, Icarus, 78, 323
Duncombe, R. L. 1969, Astronomical Papers Prepared for the Use of the American Ephemeris and Nautical Almanac, Vol. 20, Part II
Herget, P. 1947, Minor Planet Circ., 1
Hilton, J. L. 1997, AJ, 114, 402
Hilton, J. L., Seidelmann, P. K., & Middour, J. 1996, AJ, 112, 2319
Kuzmanoski, M. 1995, in IAU Symp. 172, Dynamics, Ephemerides, and Astrometry in the Solar System, ed. S. Ferraz-Mello & J.-E. Arlot (Dordrecht: Kluwer), 207
Lambert, J. V. 1985, Ph.D. thesis, New Mexico State Univ.
Landgraff, W. 1992, in IAU Symp. 152, Chaos, Resonance, and Collective Dynamical Phenomena in the Solar System, ed. S. Ferraz-Mello (Dordrecht: Kluwer), 179
Magnuson, P. 1986, Icarus, 68, 1
Marsden, B. 1991a, Minor Planet Circ., 17, 474
———. 1991b, Minor Planet Circ., 18, 847
Merline, W. J., Stern, S. A., Binzel, R. P., Festou, M. C., Flynn, B. C., & Lebofsky, L. A. 1996, BAAS, 28, 1025
Millis, R. L., et al. 1987, Icarus, 72, 507
Minor Planet Center. 1997, Minor Planet Center Observation Retrieval System
Ostro, S. J. 1993, Rev. Mod. Phys., 65, 1235
Ostro, S. J., et al. 1991, AJ, 102, 1490
Schubart, J. 1976, A&AS, 26, 405
Standish, E. M., Jr. 1986, in IAU Symp. 114, Relativity in Celestial Mechanics and Astrometry, ed. J. Kovalevsky & V. A. Brumberg (Dordrecht: Reidel), 71
———. 1987, Jet Propulsion Laboratory Interoffice Memorandum IOM 314.6-799
———. 1990, A&A, 233, 252
Standish, E. M., Jr., & Hellings, R. W. 1989, Icarus, 80, 326
Standish, E. M., Jr., Newhall, X X, Williams, J. G., & Folkner, W. M. 1995, Jet Propulsion Laboratory Interoffice Memorandum IOM 314.10-127
Stone, R. C. 1997, AJ, 113, 2317
Tedesco, E. F. 1989, in Asteroids II, ed. R. P. Binzel, T. Gehrels, & M. S. Matthews (Tucson: Univ. Arizona Press), 1090
———. 1992, *IRAS* Minor Planet Survey, Astronomical Data Center, holding 2190
Tholen, D. J. 1989, in Asteroids II, ed. R. P. Binzel, T. Gehrels, & M. S. Matthews (Tuscon: Univ. Arizona Press), 1139
Thomas, P. C., Binzel, R. P., Gaffey, M. J., Storrs, A. D., Wells, E. N., & Zellner, B. H. 1997, Science, 277, 1492
Viateau, B. 1995, Ph.D. thesis, Obs. de Paris
Viateau, B., & Rapport, M. 1997, A&A, 320, 652
Veverka, J., et al. 1997, BAAS, 29, 958
Williams, J. G. 1984, Icarus, 57, 1
———. 1989, in Asteroids II, ed. R. P. Binzel, T. Gehrels, & M. S. Matthews (Tucson: Univ. Arizona Press), 1034